2011

TRIAL

HIGHER SCHOOL CERTIFICATE

GIRRAWEEN HIGH SCHOOL

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
- Board – approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question

Total marks – 120

Attempt Questions 1 – 10

All questions are of equal value

Question 1 *(12 marks)* **Marks**

(a) Simplify $2x^2 + 3x - x(2x - 1)$ **2**

(b) Solve for x :

(i) $|2x - 3| \leq 7$ **2**

(ii) $\frac{x}{3} + \frac{2x-1}{4} = 24$ **2**

(c) Find the exact value of a and b if $a + b\sqrt{2} = \frac{3}{2\sqrt{2}-1}$ **2**

(d) Solve for x: $2x^2 - x - 3 = 0$ **2**

(e) Find the value of $\frac{\pi^2+1}{\sqrt{2}-1}$. Answer correct to three significant figures. 2

Question 2 *(12 marks)*

(a) Differentiate with respect to *x:*

(i) $x^2 e^{5x}$ **2**

(ii) $\frac{\sin x}{x^2+1}$ **2**

(iii) $(lnx)^2$ **2**

(b) Find

(i) $\int x^3 - 4x + 1. dx$ **2**

(ii) $\int_0^{\frac{\pi}{12}} cos3x. dx$ **2**

(iii) $\int \sqrt{3x - 1}\,. dx$ **2**

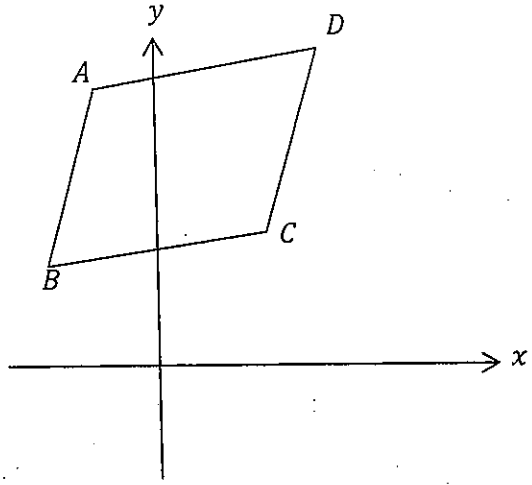
Examination continues on the next page

Question 3 *(12 Marks)* **Marks**

(a) Solve for $\theta, 0 \leq \theta \leq\leq 2\pi$: $2\sin\theta = \sqrt{3}$ **2**

(b) A quadrilateral is formed with vertices $A\ (-1{,}4) B\ (-2{,}1) C\ (1{,}2)$ and $D\ (2{,}5)$ (See below)

DIAGRAM NOT TO SCALE



(i) Find the midpoints of AC and BD. **2**

(ii) Hence or otherwise state why ABCD is a parallelogram. **1**

(iii) Find the gradients of AC and BD. **2**

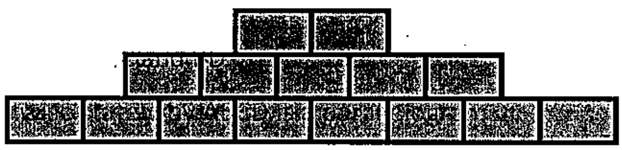
(iv) Hence or otherwise state why ABCD is a rhombus. **1**

(v) Find the lengths of AC and BD. **2**

(vi) Hence or otherwise find the area of rhombus ABCD. **2**

Question 4 *(12 marks)*

(a) A brick monument is to be built as below, with 2 bricks in the top row, 5 in the second top row, 8 in the third row and so on:



(i) How many bricks will it take to fill 20 rows? **2**

(ii) There are 1000 bricks available. How many complete rows can be made? **3**

Question 4 comtinues on the next page

(b) $ABCD$ is a square. $AP = BQ$ and $\angle PQR = 90^o$.

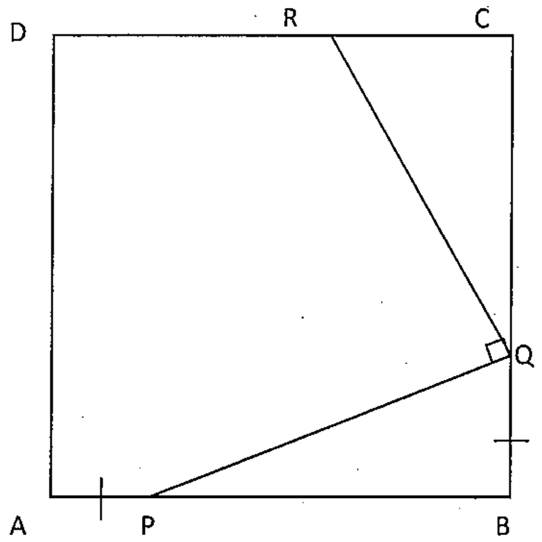


DIAGRAM NOT TO SCALE

(i) Prove $\Delta PBQ \equiv \Delta QCR$ **2**

(ii) Hence prove $AP = CR$. **1**

(c) $ABCD$ is a quadrilateral with $AB = CD$ and $\angle ABC = \angle DCB$.

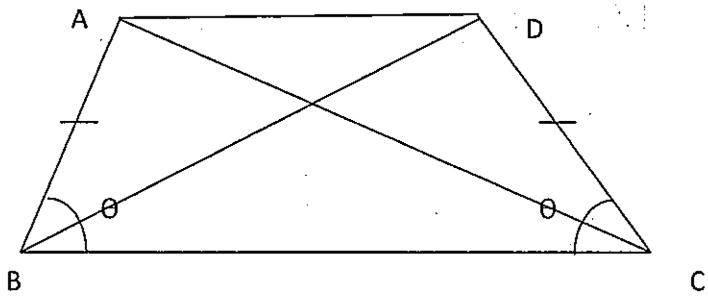


DIAGRAM NOT TO SCALE

(i) Prove $\Delta ABC \equiv \Delta DCB$. **1**

(ii) Hence OR otherwise prove $\Delta BAD \equiv \Delta CDA$. **1**

(iii) Hence or otherwise prove $ABCD$ is a trapezium. **2**

Examination continues on the next page

Question 5 *(12 Marks)* **Marks**

(a) Find the equation of the tangent to the curve $y = e^{2x}$ at the point where $x = 1$. **3**

(b) For the curve $f(x) = 3x^2 - x^3$

(i) Find the co-ordinates of the points of intersection with the x and y axes. **2**

(ii) Find the co-ordinates of the stationary points of $f(x) = 3x^2 - x^3$ and determine their nature. **4**

(iii) Find the point of inflexion of $f(x) = 3x^2 - x^3$ **1**

(iv) Sketch the graph of $f(x) = 3x^2 - x^3$ showing all stationary points, points of inflexion and intercepts with the x and y axes. **2**

Question 6 *(12 Marks)*

(a) Solve for x where x is a real number: $e^{2x} - 2e^x - 3 = 0$ **3**

(b) Find the equation of $f(x)$ if $f'(x) = 6\cos 2x$ and $f(x) = \frac{3\sqrt{2}}{2}$ when $x = \frac{5\pi}{8}$. **2**

(c) A particle is moving along a straight line so that its velocity at any time t is given by $v = t^2 - 4t$. If the particle is initially four metres to the right of $x = 0$:

(i) Find the expression for acceleration in terms of time. **1**

(ii) Find the minimum velocity of the particle. **2**

(iii) Find an expression for the displacement x in terms of time. **2**

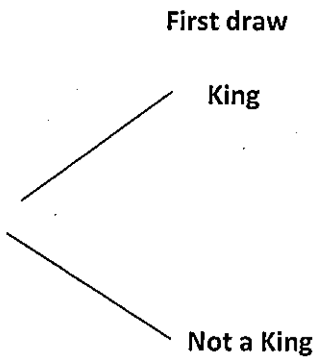
(iv) Find the total *distance* the particle travels in the first five seconds. **2**

Examination continues on the next page

Question 7 *(12 marks)* **Marks**

(a) From a standard deck of cards (4 cards of each kind, 13 of each suite 52 cards in total), 3 cards are drawn without replacement.

(i) Complete the probability tree below on your own paper displaying the probability of drawing kings from a standard deck without replacement: **2**



(ii) Find the probability of drawing 2 kings in the 3 draws. **2**

(b) Use Simpson's Rule with 4 subintervals to find an approximation for $\int_0^2 3^x . dx$ **2**

(c) 100 goats were released on to an island at the start of 1850 to provide food for shipwrecked sailors. By 1855 there were 180 goats on the island. Assuming the goat population since 1850 to follow the rule $P = Ae^{kt}$ where t is the time in years since 1850:

(i) Find the values of A and k. **3**

(ii) Find the expected goat population in 1860. **1**

(iii) Once there are more than 1000 goats on the island they start to eat more vegetation than the island can regenerate. In what year did this occur? **2**

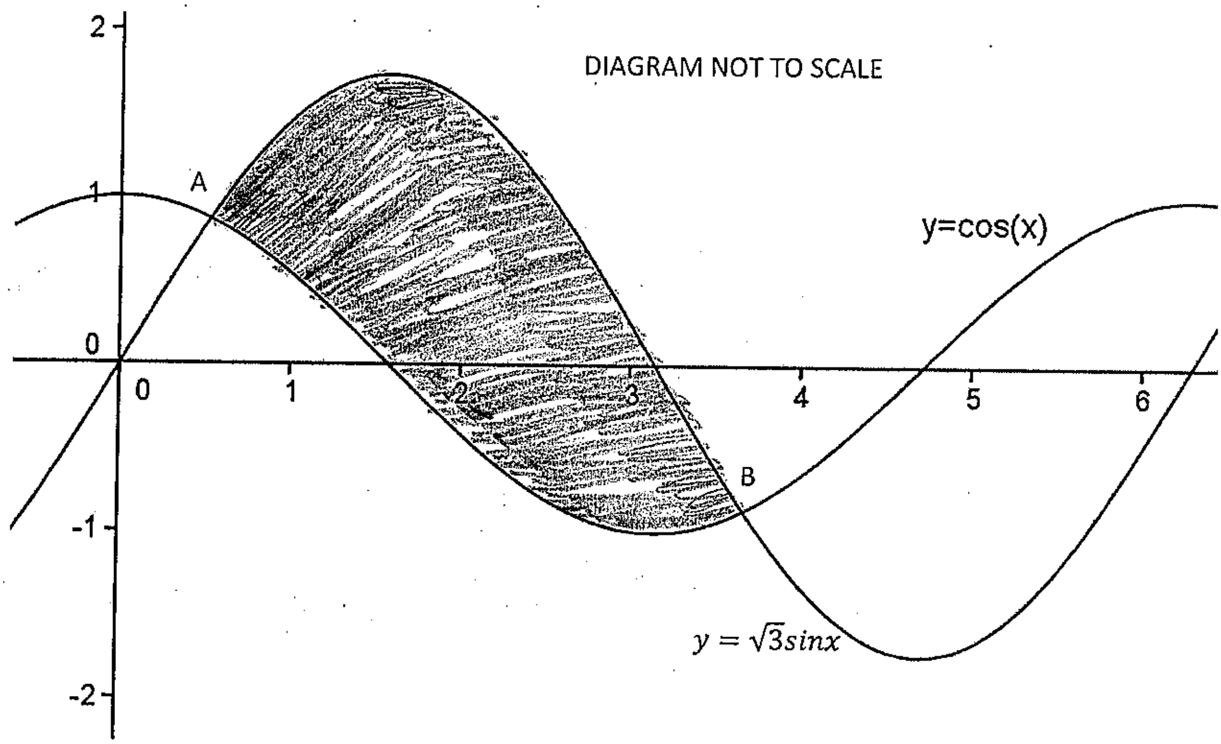
Examination continues on the next page

Question 8 *(12 Marks)* **Marks**

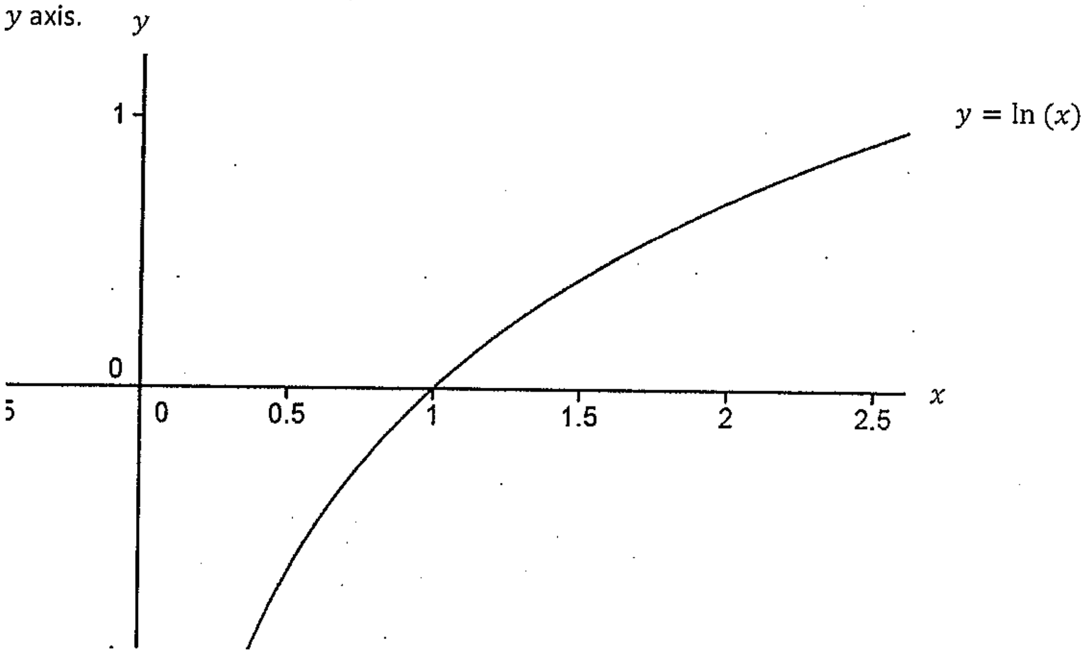
(a) (i) Solve the equation $\cos x = \sqrt{3}\sin x$ for $0 \leq x \leq 2\pi$. **3**

(ii)Hence find the shaded area between the points A and B **2**

(the intersection points of $y = \cos x$ and $y = \sqrt{3}\sin x$)

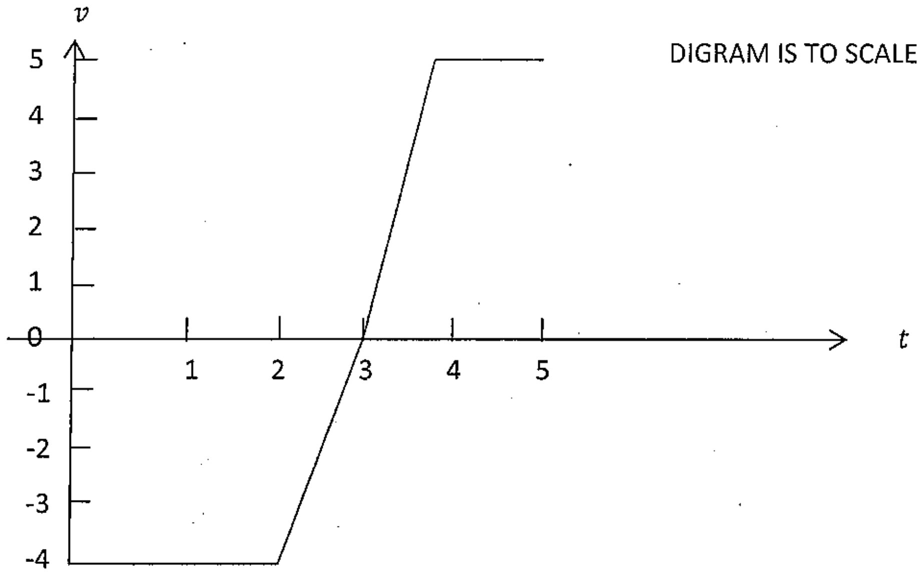


(b) Find the volume of the solid of revolution formed when the area between the curve $y = \ln(x)$, the y axis and the lines $y = 0$ and $y = 1$ is rotated around the y axis. **4**



Question 8 (continued) **Marks**

(c) The graph below shows the velocity of a particle at time t seconds:



If the particle started 2 metres to the right of $x = 0$ where did it end up after 5 seconds? **3**

Question 9 *(12 marks)*

(a) Rebecca invests \$8000 per year in a superannuation fund which pays 9%P.A. interest.

(i) Show that the amount Rebecca has in her fund after 3 years Is $\frac{\$8000 \times 1.09(1.09^3 - 1)}{0.09}$ **2**

(ii) How much will Rebecca have in her superannuation fund after 20 years? **1**

(iii) At the rate at which Rebecca is investing, how many years will it take for her superannuation fund to be worth \$1000 000? **2**

Examination continues on the next page

Question 9 (continued) **Marks**

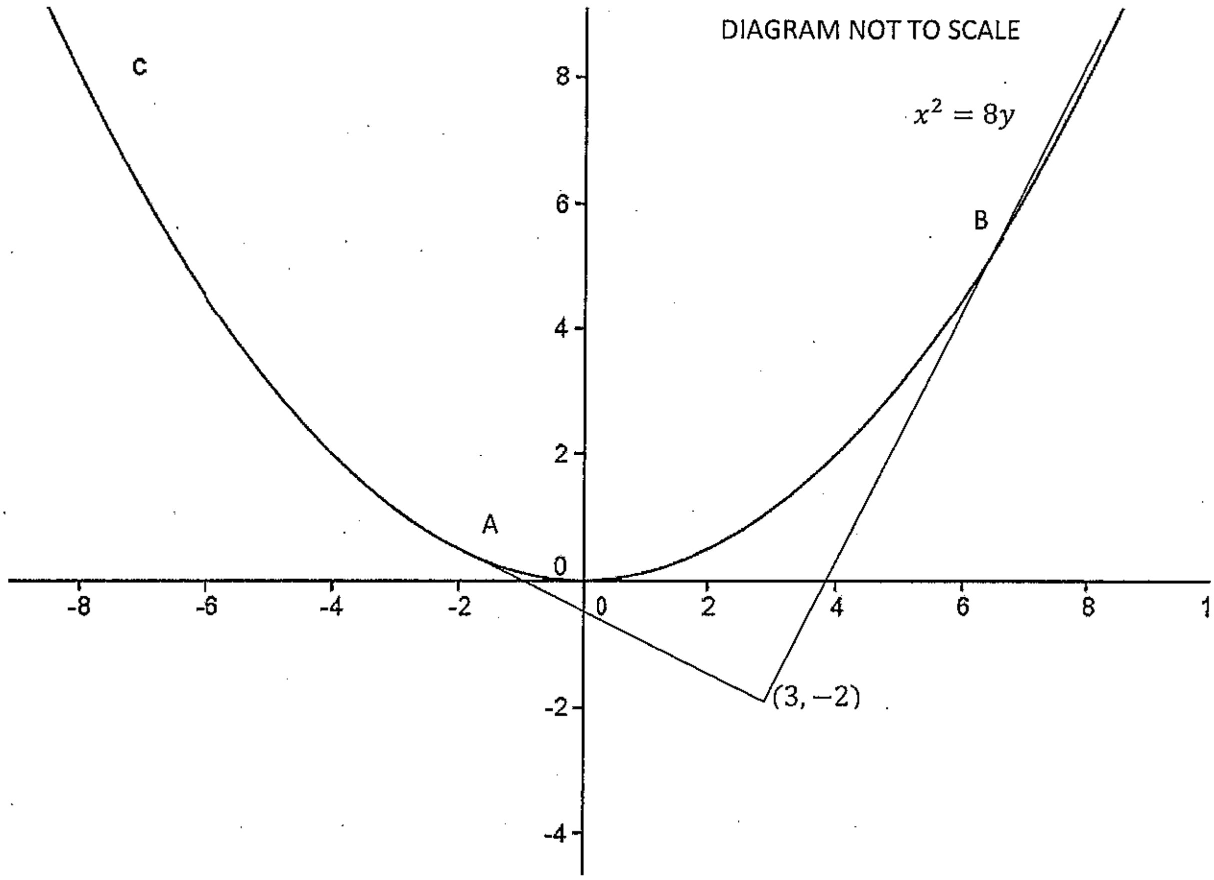
(b) (i) Show that the equation of a line through the point $(3, -2)$ with gradient m is given by $mx - y - 3m - 2 = 0$. **1**

(ii) Show that if a line through $(3, -2)$ with gradient m is a tangent to the parabola $x^2 = 8y$ then **3**

$64m^2 - 96m - 64 = 0$.

(iii) Find the two points A and B on $x^2 = 8y$ where the tangents from $(3, -2)$ meet the parabola $x^2 = 8y$. **2**

(iv) Show that AB is a focal chord. **1**



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Question 10 (12 marks) **Marks**

(a) Haasika is paying off a home loan of \$500 000 over 20 years in equal monthly instalments at a rate of 7.2%P.A. If she is paying off the loan at \$P per month:

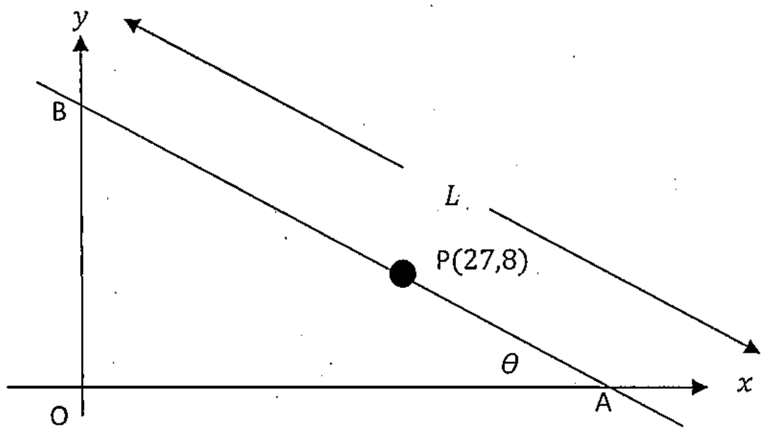
(i) Show that the amount she has left to repay after 3 months is given by **1**

$A = \$500\,000 \times 1.006^3 - P(1 + 1.006 + 1.006^2)$

(ii) Find an expression for the amount Haasika has left to repay after n months. **2**

(iii) Find the amount of each monthly repayment. Answer correct to the nearest dollar. **1**

(b) In the diagram below, a straight line passes through the fixed point P (27,8) and meets the x and y axes at A and B respectively. L is the distance along the line AB and the acute angle $\angle OAB = \theta$.



(i) Show that $L = \frac{27}{\cos\theta} + \frac{8}{\sin\theta}$ **2**

(ii) Show that $\frac{dL}{d\theta} = \frac{27sin^3\theta - 8cos^3\theta}{sin^2\theta cos^2\theta}$ **1**

(iii) Hence show that L is a minimum when $\tan\theta = \frac{2}{3}$. **3**

(iv) Find the minimum distance between A and B. **2**

END OF EXAMINATION

2 Unit Maths p.1

Girraween HS Trial 2011

— Solutions —

Q.(1)(a) $2x^2+3x-x(2x-1)$

$=2x^2+3x-2x^2+x$

$=4x$

(b)(i) $|2x-3|\leq 7$

$-7\leq 2x-3\leq 7$

$-4\leq 2x\leq 10$

$-2\leq x\leq 5$

(ii) $\frac{x}{3}+\frac{2x-1}{4}=24$ $\times 12$ $\times 12$

$4x+6x-3=288$

$10x-3=288$

$10x=291$

$x=29{\cdot}1$ or $29\frac{1}{10}$

(c) $a+b\sqrt{2}=\frac{3}{2\sqrt{2}-1}\times\frac{(2\sqrt{2}+1)}{(2\sqrt{2}+1)}$

$=\frac{6\sqrt{2}+3}{8-1}$

$a+b\sqrt{2}=\frac{6}{7}\sqrt{2}+\frac{3}{7}$

$\therefore a=\frac{3}{7},\ b=\frac{6}{7}$

(d) $2x^2-x-3=0$

$(2x-3)(x+1)=0$

$x=\frac{3}{2}$ or $x=-1$.

(e) $\frac{\pi^2+1}{\sqrt{2}-1}=$

$=26{\cdot}2415\ldots$

$\doteqdot 26{\cdot}2$ (to 3 significant figures).

Girraween HS 2 Unit Maths HSC Trial 2011 Solutions p.2

Q.(2)(a)(i) $y=x^2e^{5x}$

$\frac{dy}{dx}=u'v+v'u$

$=2xe^{5x}+5e^{5x}\times x^2$

$=xe^{5x}(2+5x)$

(ii) $y=\frac{\sin x}{x^2+1}$

$\frac{dy}{dx}=\frac{u'v-v'u}{v^2}$

$=\frac{\cos x(x^2+1)-2x\sin x}{(x^2+1)^2}$

$\frac{dy}{dx}=\frac{x^2\cos x+\cos x-2x\sin x}{(x^2+1)^2}$

(iii) $y=[\ln(x)]^2$. Letting $u=\ln x$

$\frac{du}{dx}=\frac{1}{x}$

$y=u^2$

$\frac{dy}{du}=2u$

$=2\ln x$

$\frac{dy}{dx}=\frac{dy}{du}\times\frac{du}{dx}$

$=2\ln x\times\frac{1}{x}$

$\frac{dy}{dx}=\frac{2\ln x}{x}$

(b)(i) $\int x^3-4x+1\,dx$

$=\frac{1}{4}x^4-2x^2+x+C$

(ii) $\int_0^{\frac{\pi}{12}}\cos 3x\,dx$

$=\left[\frac{1}{3}\sin 3x\right]_0^{\frac{\pi}{12}}$

$=\frac{1}{3}\sin\frac{\pi}{4}-\frac{1}{3}\sin 0$

$=\frac{\sqrt{2}}{6}$

(iii) $\int\sqrt{3x-1}\,dx$

$=\frac{1}{3\times\frac{3}{2}}(3x-1)^{\frac{3}{2}}+C$

[using $\int(ax+b)^n\,dx=\frac{1}{a(n+1)}(ax+b)^{n+1}+C$]

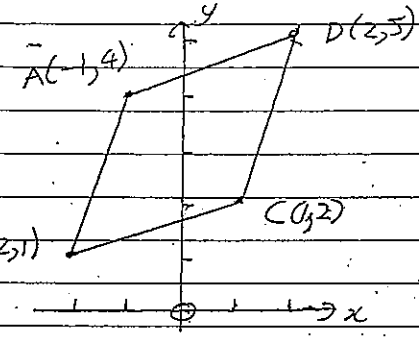
$=\frac{2}{9}(3x-1)\sqrt{3x-1}+C$

Q.(3)(a) $2\sin\theta = \sqrt{3}$

$\sin\theta = \frac{\sqrt{3}}{2}$

$\theta = \frac{\pi}{3}$ or $\theta = \frac{2\pi}{3}$.

(b)



(i) Midpoint AC

$= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$= \left(\frac{-1+1}{2}, \frac{4+2}{2}\right)$

$= (0, 3)$.

Similarly, midpoint BD

$= \left(\frac{-2+2}{2}, \frac{1+5}{2}\right)$

$= (0, 3)$

(ii) ABCD is a parallelogram [diagonals bisect each other].

(iii) $mAC = \frac{y_2-y_1}{x_2-x_1}$

$= \frac{2-4}{1--1}$

$mAC = -1$

Similarly, mBD

$= \frac{5-1}{2--2}$

$mBD = 1$

(iv) $mAC \times mBD$

$= -1 \times 1$

$= -1$

Hence $AC \perp BD$

$\therefore$ ABCD is a rhombus [diagonals $\perp$ and bisect each other]

(v) Length $AC = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{(1+1)^2+(2-4)^2}$

$= \sqrt{8}$ units.

Similarly, Length BD

$= \sqrt{(2+2)^2+(5-1)^2}$

$= \sqrt{32}$ units.

(vi) Area rhombus ABCD $= \frac{1}{2}xy$ [x, y diagonals]

$= \frac{1}{2}\times\sqrt{8}\times\sqrt{32}$

$= 8$ square units

Q.(4)(a)(i) N° of bricks = 2 + 5 + 8 + ...

$\rightarrow$ Arithmetic series: $a=2$, $d=3$, $n=20$ [for 1st 20 rows]

By $S_n = \frac{n}{2}(2a+(n-1)d)$

$S_{20} = \frac{20}{2}(2\times2+19\times3)$

$= 610$

$\therefore$ It would take 610 bricks to fill 20 rows.

(ii) If $S = 1000$

$S_n = \frac{n}{2}[2a+(n-1)d]$

$1000 = \frac{n}{2}[2\times2+(n-1)\times3]$

$1000 = \frac{n}{2}[3n+1]$.

$2000 = 3n^2+n$

$0 = 3n^2+n-2000$

Using the quadratic formula,

$n = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$= \frac{-1\pm\sqrt{1^2-4\times3\times-2000}}{2\times3}$

$n = \frac{-1\pm\sqrt{24001}}{6}$

$\therefore n = \frac{-1+\sqrt{24001}}{6}$ or $n = \frac{-1-\sqrt{24001}}{6}$

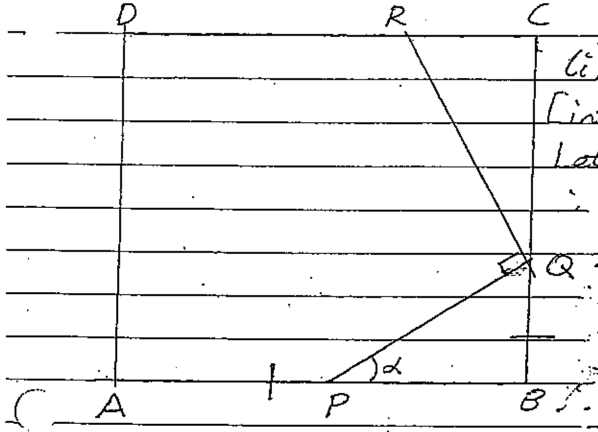
$n = 25.6...$ or $n = -25.987...$

As n is a number of rows, it must be positive

$\therefore n = 25$. [the 26th row would take 1027 bricks $\rightarrow$ too many].

$\therefore$ 25 rows can be made [complete] using 1000 bricks.

Q.(4)(b)



(i) $\angle PBQ = \angle QCR = 90°$ [interior $\angle$'s of square ABCD].

Let $\angle QPB = \alpha$

$\therefore \angle PQB = 90° - \alpha$ [$\angle$ sum ΔPBQ]

$\angle RQC = \alpha$ [$\angle$'s in straight line BQC]

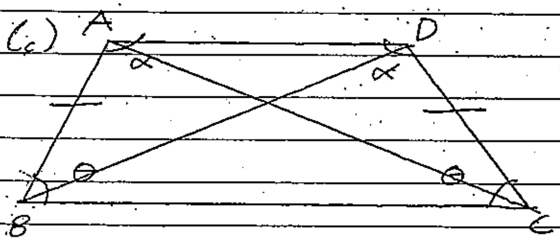
$\therefore \Delta PBQ \equiv \Delta QCR$ [AAS]

(ii) Hence $CR = BQ$ [matching sides, $\Delta PBQ \equiv \Delta QCR$].

$CR = AP$ [data $\to BQ = AP$].

$\therefore CR = AP$

(c)



(i) In Δ's ABC & DCB

BC is common

$AB = DC$ [data]

$\angle ABC = \angle DCB$ [data]

$\therefore \Delta ABC \equiv \Delta DCB$ [SAS].

(ii) In Δ's BAD and CDA

$BD = CA$ [matching sides in $\equiv \Delta$'s, ΔABC & ΔDCB].

$BA = CD$ [data — don't need to restate this]

AD common

$\therefore \Delta BAD \equiv \Delta CDA$ [SSS]

(iii) Let $\angle BAD = \alpha$

$\angle CDA = \alpha$ [matching $\angle$'s, $\Delta BAD \equiv \Delta CDA$]

$\therefore \angle$ sum quadrilateral ABCD $= 2\alpha + 2\theta$.

$2\alpha + 2\theta = 360°$ [$\angle$ sum quadrilateral ABCD]

$\alpha + \theta = 180°$.

$\therefore AD \parallel BC$ [co-interior $\angle$'s supplementary]. $\Rightarrow$ ABCD is a trapezium.

Q.(5)(a) $y = e^{2x}$

When $x = 1$, $y = e^2$.

$\frac{dy}{dx} = 2e^{2x}$

When $x=1$, $\frac{dy}{dx} = 2e^2$

By $y - y_1 = m(x - x_1)$

$y - e^2 = 2e^2(x-1)$

$y - e^2 = 2e^2x - 2e^2$

$y = 2e^2x - e^2$

(b)(i) y intercept = 0

x intercepts: $y = 0$

$3x^2 - x^3 = 0$

$x^2(3-x) = 0$

x intercepts = 0 & 3.

(ii) $f'(x) = 6x - 3x^2$

If $f'(x) = 0$, $6x - 3x^2 = 0$

$3x(2-x) = 0$

$x = 0$ or $x = 2$

When $x = 0$, $f(x) = 0$ [y intercept]

When $x = 2$, $f(x) = 3 \times 2^2 - 2^3$

$= 4$

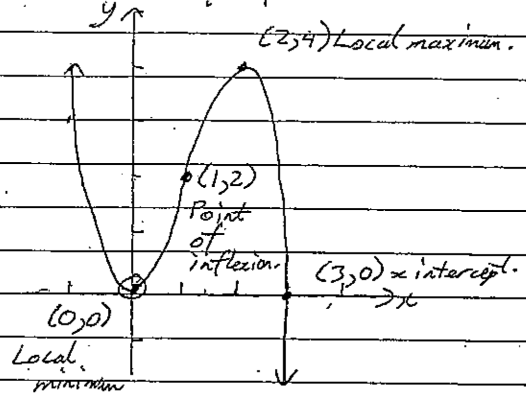
Nature of turning points: $f''(x) = 6 - 6x$.

At $(0,0)$ $f''(x) = 6 - 6\times 0 = 6 \Rightarrow$ Local MINIMUM at $(0,0)$

At $(2,4)$ $f''(x) = 6 - 6\times 2 = -6 \Rightarrow$ Local MAXIMUM at $(2,4)$

(iii) Point of inflexion: $f''(x) = 0$ $6 - 6x = 0$ $x = 1$. At $x = 1$, $f(x) = 3\times 1^2 - 1^3 = 2$.	$f''(x)$ DOES change sign at this point as $f''(0) > 0$ and $f''(2) < 0$ [from Part (ii)] $\therefore (1,2)$ IS a point of inflexion.

Q.(5)(b) [continued]: Sketch of $f(x)=3x^2-x^3$:



Q.(6)(a) $e^{2x}-2e^{x}-3=0$

Letting $a=e^{x}$,

$a^2-2a-3=0$

$(a-3)(a+1)=0$

$a=3$ or $a=-1$.

As $a=e^{x}$,

$e^{x}=3$ or $e^{x}=-1$.

$x=\ln 3$ For $e^{x}=-1$ there are no real solutions.

Q.(6)(c) $f(x)=\int 6\cos 2x\,dx$

$=3\sin 2x+C$

If $f(x)=\frac{3\sqrt{2}}{2}$ when $x=\frac{7\pi}{8}$

$\frac{3\sqrt{2}}{2}=3\sin\frac{5\pi}{4}+C$

$\frac{3\sqrt{2}}{2}=\frac{-3\sqrt{2}}{2}+C$

$3\sqrt{2}=C$

$\therefore f(x)=3\sin 2x+3\sqrt{2}.$

Q.(6)(b)(i) $v=t^2-4t$

$a=2t-4$

(ii) Minimum velocity: $a=0$

$2t-4=0$

$t=2.$

v at this time $=2^2-4\times 2$

$=-4$ m/s.

Checking that this is a minimum:

When $t=1$, $v=1^2-4\times 1=-3$ m/s] Both >-4 m/s

When $t=3$, $v=3^2-4\times 3=-3$ m/s so minimum velocity at $t=2$.

$\Rightarrow$ Could also argue that $v=t^2-4t$ is a U [Concave up] parabola so only turning point is a local minimum.

(iii) $x=\int t^2-4t\,dt$

$=\frac{1}{3}t^3-2t^2+C$

As $x=4$ when $t=0$, $-2\times 0^3+C=4 \Rightarrow C=4.$

$\therefore x=\frac{1}{3}t^3-2t^2+4.$

(iv) Need to find when $v=0$:

$t^2-4t=0$

$t(t-4)=0$

$t=0$ or $t=4$

$\therefore$ Particle goes backwards [negative velocity] from $t=0$ to $t=4$.

When $t=4$, $x=\frac{1}{3}\times 4^3-2\times 4^2+4$

$=-6\frac{2}{3}$

$\Rightarrow$ In 1ST 4 seconds, particle travels from 4 to $-6\frac{2}{3}=10\frac{2}{3}$ m.

Particle goes FORWARDS from $t=4$ to $t=5$.

When $t=5$, $x=\frac{1}{3}\times 5^3-2\times 5^2+4$

$=-4\frac{1}{3}$

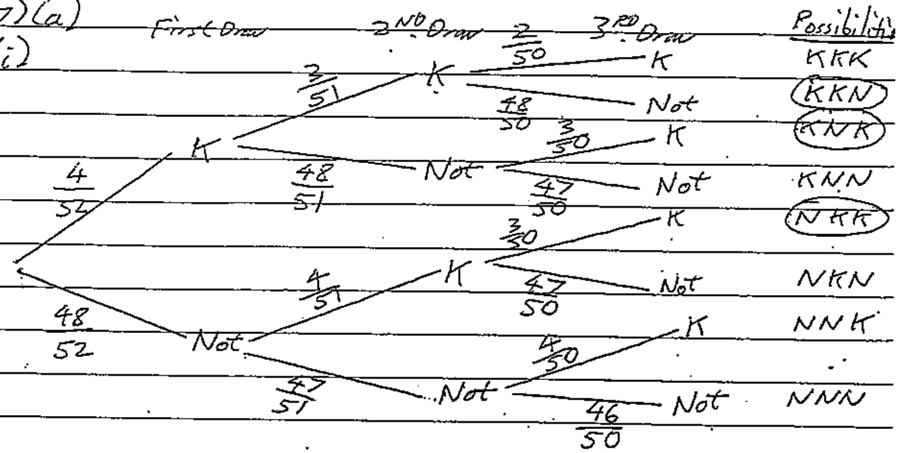
$\Rightarrow$ In 5TH second particle travels $2\frac{1}{3}$ m RIGHT.

$\therefore$ Total distance travelled $=13$ m in 1ST 5 seconds.

Note: Could also do $\left|\int_0^4 t^2-4t\,dt\right|+\int_4^5 t^2-4t\,dt$.

Q.(7)(a)

(i)

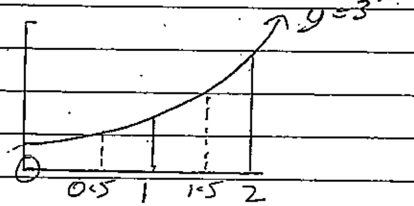


(ii) Probability of drawing 2 Kings in the 3 draws

$= 3\times\left(\frac{4}{52}\times\frac{3}{51}\times\frac{48}{50}\right)$

$= \frac{72}{5525}.$

(b)



$A = \frac{1}{3}h\left[y_0 + 4(y_1+y_3) + 2y_2 + y_4\right]$

$= \frac{1}{3}\times 0.5\times\left[3^0 + 4\left(3^{0.5}+3^{1.5}\right) + 2\times 3^1 + 3^2\right]$

$= 7.2854\ldots$

$\doteqdot 7.29$ square units (2 DP).

Q.(7)(c)(i) When $t=0$, $P=100$

$\therefore\ 100 = Ae^0$

$100 = A.$

When $t=5$, $P=180$

$\therefore\ 180 = 100e^{5k}$

$1.8 = e^{5k}$

$\ln(1.8) = 5k$

$\frac{\ln(1.8)}{5} = k \quad (\doteqdot 0.11755\ldots)$

(ii) Expected goat population in 1860: $(t=10)$

$P = 100e^{kt}$

$\therefore$ When $t=10$, $P = 100e^{10\times\frac{\ln(1.8)}{5}}$

$= 324$ goats.

(iii) Year when $P=1000$

$1000 = 100e^{kt}$

$10 = e^{kt}$

$\ln(10) = kt,\ k = \frac{\ln(1.8)}{5}$

$\therefore\ \frac{\ln(10)}{\left(\frac{\ln 1.8}{5}\right)} = t$

$19.58\ldots = t.$

→ During 1869 the goat population will exceed 1000.

(8)(a)(i) $\cos x = \sqrt{3}\sin x$

$\div$ BS by $\sqrt{3}\cos x$

$\frac{1}{\sqrt{3}} = \tan x$

$x = \frac{\pi}{6}, \frac{7\pi}{6}$

(ii) $A = \int_{\frac{\pi}{6}}^{\frac{7\pi}{6}} \sqrt{3}\sin x - \cos x \, dx$

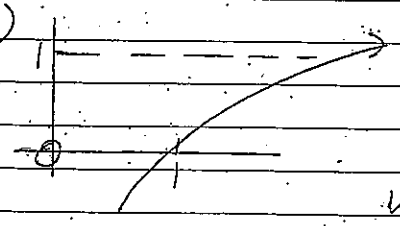
$= \left[-\sqrt{3}\cos x - \sin x\right]_{\frac{\pi}{6}}^{\frac{7\pi}{6}}$

$= \left(-\sqrt{3}\cos\frac{7\pi}{6} - \sin\frac{7\pi}{6}\right) - \left(-\sqrt{3}\cos\frac{\pi}{6} - \sin\frac{\pi}{6}\right)$

$= \frac{\sqrt{3}\times\sqrt{3}}{2} + \frac{1}{2} - \left(-\frac{\sqrt{3}\times\sqrt{3}}{2} - \frac{1}{2}\right)$

$=$ 4 square units.

(b)



As $y = \ln(x)$,

$x = e^y$.

$V = \pi\int_0^1 x^2 \, dy$

$= \pi\int_0^1 e^{2y} \, dy$

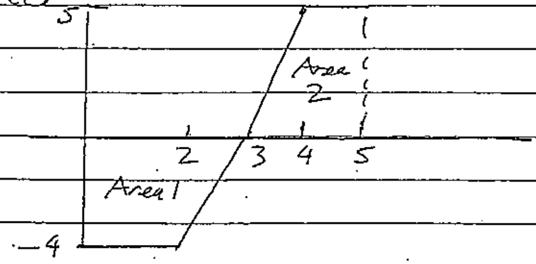
$= \pi\left[\frac{1}{2}e^{2y}\right]_0^1$

$= \pi\left[\frac{1}{2}e^2 - \frac{1}{2}e^0\right]$

$= \frac{\pi}{2}\times(e^2 - 1)$

$V = \frac{\pi}{2}(e^2-1) \doteq 10.04$ cubic units

Q. (8)(c)



Distance travelled = area under graph.

Area 1 $= \frac{1}{2}h(a+b)$

$= \frac{1}{2}\times 4\times(3+2)$

$= 10.$

~~$\Rightarrow$ Particle initially travels 10m backwards.~~

Area 2 $= \frac{1}{2}\times 5\times(2+1)$

$= 7.5$

$\Rightarrow$ Particle then travels 7.5m forwards.

$\therefore$ After 5 seconds, particle is at

$2 - 10 + 7.5 = -0.5$

$= 0.5$m to the LEFT of 0.

Q.(9)(a)(i) Amount in account:

Year	Start of Year	End of Year × amount at start by 1·09
1	\$8000	\$8000×1·09
2	\$8000+\$8000×1·09	\$8000×1·09+\$8000×1·09^2
3	\$8000+\$8000×1·09+\$8000×1·09^2	\$8000×1·09+\$8000×1·09^2 +\$8000×1·09^3

∴ Amount after 3 years $= \$8000(1{\cdot}09+1{\cdot}09^2+1{\cdot}09^3)$

Using $S_n = \dfrac{a(r^n-1)}{r-1}$

$$1{\cdot}09+1{\cdot}09^2+1{\cdot}09^3 = \frac{1{\cdot}09(1{\cdot}09^3-1)}{1{\cdot}09-1}$$
$$= \frac{1{\cdot}09(1{\cdot}09^3-1)}{0{\cdot}09}$$

∴ Total after 3 years $= \dfrac{\$8000\times1{\cdot}09(1{\cdot}09^3-1)}{0{\cdot}09}$

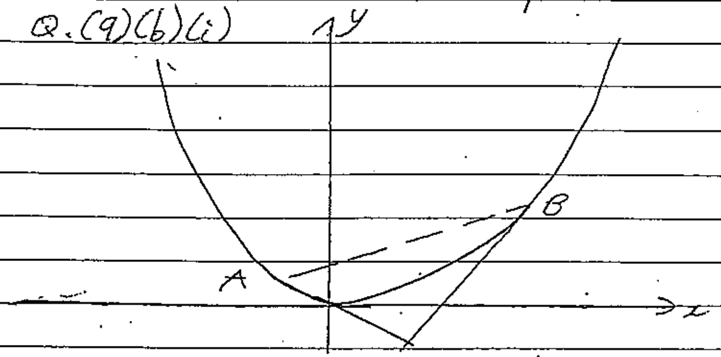
(ii) After 20 years, Rebecca will have
$$\frac{\$8000\times1{\cdot}09(1{\cdot}09^{20}-1)}{0{\cdot}09}$$
$= \$446\ 116{\cdot}24$ in her account.

(iii) $\dfrac{\$8000\times1{\cdot}09(1{\cdot}09^n-1)}{0{\cdot}09} = 1\,000\,000$ $\quad \times\dfrac{0{\cdot}09}{8000\times1{\cdot}09}$

$$1{\cdot}09^n-1 = 10{\cdot}321..$$
$$1{\cdot}09^n = 11{\cdot}321..$$
$$\ln(1{\cdot}09^n) = \ln(11{\cdot}321)$$
$$n\ln(1{\cdot}09) = \ln(11{\cdot}321..)$$
$$n = \frac{\ln(11{\cdot}321..)}{\ln 1{\cdot}09}$$
$$n = 28{\cdot}15..$$

∴ It will take 28·15 ≈ 29 years for super to reach 1000 000.

Q.(9)(b)(i)



By $y-y_1=m(x-x_1)$
$$y+2=m(x-3)$$
$$y+2=mx-3m$$
$$0=mx-y-3m-2$$
$$\therefore mx-y-3m-2=0$$

(ii) If $mx-y-3m-2=0$ is a tangent to $x^2=8y$ then there is only 1 solution when $mx-y-3m-2=0$ & $x^2=8y$ are solved simultaneously.

$mx-y-3m-2=0$ (1)

$x^2=8y$ (2) Sub.(2) in (1):

$mx-\frac{1}{8}x^2-3m-2=0$ $\quad\times8$

$8mx-x^2-24m-16=0$

$x^2-8mx+24m+16=0$ (A) ⇒ Quadratic equation.

If a tangent then $\Delta=0$
$$(-8m)^2-4\times1\times(24m+16)=0$$
$$64m^2-96m-64=0$$

Q.(9)(b)(iii) $64m^2 - 96m - 64 = 0$

$2m^2 - 3m - 2 = 0$

$(2m+1)(m-2) = 0$

$m = -\frac{1}{2}$ or $m = 2$.

If $m = -\frac{1}{2}$, x value where

$mx - y - 3m - 2 = 0$ &

$x^2 = 8y$

can be solved simultaneously.

Sub. into equation (A): from Part (ii)]

$x^2 - 8mx + 24m + 16 = 0$

$x^2 - 8 \times -\frac{1}{2}x + 24 \times -\frac{1}{2} + 16 = 0$

$x^2 + 4x + 4 = 0$

$(x+2)^2 = 0$

$x = -2$.

As $x^2 = 8y$, $(-2)^2 = 8y$

$\frac{1}{2} = y$.

$\therefore$ Point $A = (-2, \frac{1}{2})$.

If $m = 2$, x value where

$mx - y - 3m - 2 = 0$ &

$x^2 = 8y$ can be solved simultaneously

$x^2 - 8mx + 24m + 16 = 0$

$x^2 - 8 \times 2x + 24 \times 2 + 16 = 0$

$x^2 - 16x + 64 = 0$

$(x-8)^2 = 0$

$x = 8 \Rightarrow 8y$ $x^2 = 8y$, $8^2 = 8y$

$8 = y$.

Point $B = (8, 8)$

Q.(9)(b)(iv) Chord AB:

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{8 - \frac{1}{2}}{8 + 2}$

$= \frac{3}{4}$.

By $y - y_1 = m(x - x_1)$

Equation of chord AB:

$y - 8 = \frac{3}{4}(x - 8)$.

y intercept of chord AB: $[x = 0]$

$y - 8 = \frac{3}{4}(0 - 8)$

$y - 8 = -6$

$y = 2$.

y intercept of chord $AB = (0, 2)$.

Focus of parabola $x^2 = 8y$

By $x^2 = 4ay$,

$4a = 8 \Rightarrow a = 2$.

$\therefore$ Focus $= (0, a) = (0, 2)$

$\therefore$ Chord AB is a focal chord.

Q. (10)(a)(i) Amount left to repay

Month	Start of Month	End
1	\$500 000	$\$500\,000 \times 1{\cdot}006 - P$
2		$1{\cdot}006 \times (\$500\,000 \times 1{\cdot}006 - P) - P$
		$= \$500\,000 \times 1{\cdot}006^2 - P(1 + 1{\cdot}006)$
3		$[\$500\,000 \times 1{\cdot}006^2 - P(1+1{\cdot}006)] \times 1{\cdot}006 -$
		$= \$500\,000 \times 1{\cdot}006^3 - P(1 + 1{\cdot}006 + 1{\cdot}006^2)$

Note: 7·2% P.A = 0·6% per month.

(ii) Amount left after n months

$= \$500\,000 \times 1{\cdot}006^n - P(1 + 1{\cdot}006 + 1{\cdot}006^2 + \ldots + 1{\cdot}006^{n-1})$

Using $S_n = \dfrac{a(r^n - 1)}{r-1}$

$1 + 1{\cdot}006 + \ldots + 1{\cdot}006^{n-1} = \dfrac{1(1{\cdot}006^n - 1)}{0{\cdot}006}$

$\therefore$ Amount left to repay after n months

$= \$500\,000 \times 1{\cdot}006^n - \dfrac{P(1{\cdot}006^n - 1)}{0{\cdot}006}$

(iii) After 20 years = 240 months,

Amount left to repay $= \$0$

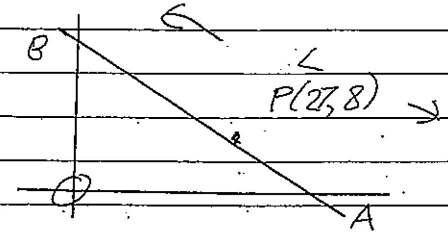
$\therefore \$500\,000 \times 1{\cdot}006^{240} - \dfrac{P(1{\cdot}006^{240} - 1)}{0{\cdot}006} = 0$

$\$500\,000 \times 1{\cdot}006^{240} = \dfrac{P(1{\cdot}006^{240} - 1)}{0{\cdot}006}$

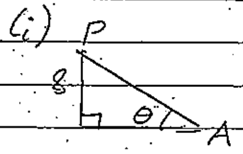
$\$3936{\cdot}75 = P$

$\Rightarrow$ Hassika's monthly repayments amount to \$3937 (per month).

Q. (10)(b)



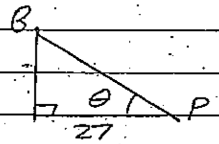
(i)



Distance PA:

$\sin\theta = \dfrac{8}{PA}$

$\therefore PA = \dfrac{8}{\sin\theta}$



Distance PB:

$\cos\theta = \dfrac{27}{PB}$

$\therefore PB = \dfrac{27}{\cos\theta}$

$\therefore L = \text{Distance } PA + \text{Distance } PB$

$= \dfrac{8}{\sin\theta} + \dfrac{27}{\cos\theta}$

(ii) $\dfrac{dL}{d\theta} = \dfrac{-8}{\sin^2\theta} \times \cos\theta - \dfrac{27}{\cos^2\theta} \times -\sin\theta$

$= \dfrac{27\sin\theta}{\cos^2\theta} - \dfrac{8\cos\theta}{\sin^2\theta}$

$= \dfrac{27\sin^3\theta - 8\cos^3\theta}{\cos^2\theta \cdot \sin^2\theta}$

(iii) L is a minimum where $\dfrac{dL}{d\theta} = 0$

i.e. $\dfrac{27\sin^3\theta - 8\cos^3\theta}{\cos^2\theta\sin^2\theta} = 0$ $\quad \times$ BS by $\cos^2\theta\sin^2\theta$

PTO →

Q.(10)(b) (continued)

(iii) $27\sin^3\theta - 8\cos^3\theta = 0$

$\div$ BS by $\cos^3\theta$, $\cos\theta \neq 0$ [won't be as θ is acute],

$27\tan^3\theta - 8 = 0$

$27\tan^3\theta = 8$

$\tan^3\theta = \frac{8}{27}$

$\tan\theta = \frac{2}{3}$

[as θ is acute, $\tan\theta$ is positive].

Checking to see if L is minimum:

If $\tan\theta = 1$ [$\theta >$ than if $\tan\theta = \frac{2}{3}$],

$$\frac{dL}{d\theta} = \frac{27\times\frac{\sqrt{2}}{4} - 8\times\frac{\sqrt{2}}{4}}{\frac{1}{4}}$$

$$= 19\sqrt{2} > 0$$

If $\tan\theta = \frac{1}{\sqrt{3}}$ [$\theta <$ than if $\tan\theta = \frac{2}{3}$].

$$\frac{dL}{d\theta} = \frac{27\times\frac{1}{8} - 8\times\frac{3\sqrt{3}}{8}}{\frac{1}{4}\times\frac{3}{4}}$$

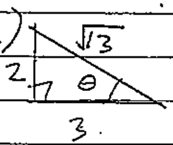
$$= \frac{\cancel{27}^{9} - \cancel{24}^{8}\sqrt{3}}{\cancel{8}_{1}} \times \frac{\cancel{16}^{2}}{\cancel{3}_{1}}$$

$$= 9 - 8\sqrt{3} < 0$$

$\therefore$ As $\frac{dL}{d\theta} < 0$ where $\theta <$ θ if $\tan\theta = \frac{2}{3}$ & $\frac{dL}{d\theta} > 0$ where $\theta >$ if $\tan\theta = \frac{2}{3}$

$\rightarrow$ Minimum where $\tan\theta = \frac{2}{3}$.

(iv)



$\tan\theta = \frac{2}{3}$ $\sin\theta = \frac{2}{\sqrt{13}}$ $\cos\theta = \frac{3}{\sqrt{13}}$

Minimum $L = \frac{8}{\sin\theta} + \frac{27}{\cos\theta}$

$$= 8\times\frac{\sqrt{13}}{2} + \frac{27\times\sqrt{13}}{3}$$

Minimum Length $L = 13\sqrt{13}$ units